

A further study of the surface zonal flow predicted by an eddy flux parametrization scheme

By G.-X. WU

Institute of Atmospheric Physics, Chinese Academy of Sciences, Peking, People's Republic of China

and

A. A. WHITE

Meteorological Office, Bracknell

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SUMMARY

The problem of calculating the surface zonal flow using Green's large-scale eddy flux parametrizations in spherical geometry is re-examined. In a 2-level, quasi-geostrophic (QG1) model, low-latitude easterlies and mid-latitude westerlies of reasonable intensity can be obtained by assuming a more realistic baroclinicity than that applied in a study reported by White in 1977. This result also depends on the use of a more realistic value for the latitude average of a certain transfer coefficient; but no detailed treatment of its spatial variation is found to be necessary. Some further solutions are obtained using a strictly inconsistent formulation in which the Coriolis parameter is allowed its true latitude variation in all terms (in contrast to the QG1 model). In line with the conclusions of White's earlier study it seems in this case that detailed specification of the relevant transfer coefficient's spatial variation would be needed in order to produce a realistic surface zonal flow. Regarding the surface flow problem as a test of Green's parametrization scheme, we conclude that the latter performs quite well if QG1 approximations are applied to the Coriolis parameter. However, further refinement of the scheme is evidently required for good performance when these approximations are not applied.

1. INTRODUCTION

One of the most important aspects of the tropospheric general circulation is the zonal average surface zonal flow U_s . It is well established that large-scale eddy fluxes of zonal angular momentum are mainly responsible for the maintenance of U_s against the effects of frictional interaction with the earth's surface—see Lorenz (1967), Kidson *et al.* (1969) and Oort and Rasmusson (1971). Figure 1 shows a typical latitude profile of U_s and the angular momentum and vorticity transfers required to support it. Clearly, the eddy motion does not act simply to transfer angular momentum down the angular velocity gradient or vorticity down the absolute vorticity gradient (Lorenz 1967; Starr 1968; Read 1986). Thus the surface zonal flow affords a stringent test of large-scale eddy parametrization schemes of the type used in statistical climate models (see Schneider and Dickinson 1974; Saltzman 1978).

Green (1970)—herein referred to as G70—proposed a mixing-type parametrization scheme which attempted to overcome the difficulties mentioned above. He suggested that the large-scale transient eddy motion acts to mix, in a readily idealizable fashion, only those quantities which are closely conserved in the quasi-geostrophic eddy evolution. Such quantities include potential temperature and potential vorticity, but not angular momentum or vorticity (neither of which obeys a Lagrangian conservation law in three-dimensional baroclinic motion). Green proposed simple expressions for the eddy fluxes of conserved quantities in terms of the relevant mean gradients and eddy diffusivities—called transfer coefficients.

By assuming a reasonable form for the zonal mean temperature field, Green used his potential vorticity mixing model to calculate a theoretical surface zonal flow variation in a β -plane formulation. His results were realistic: westerlies in middle latitudes and easterlies in extreme latitudes, with reasonable maximum values. In an extension of Green's study, White (1977) (herein denoted as W77) showed that the various assumptions of Green's analysis were uncritical so long as a realistic β effect was present.

Examination of the problem in a quasi-geostrophic spherical polar formulation, however, led to less encouraging results. It was found that geometrical effects (including the variation of absolute vorticity gradients on the sphere) conspired to make the calculated surface flow much more sensitive to the assumed behaviour of a certain transfer coefficient than was the case in the β -plane formulation. In particular, W77 concluded that fairly detailed treatment of the spatial variation of the transfer coefficient was necessary in order to obtain a surface zonal flow of realistic strength and having polar easterlies. The required treatment might indeed go beyond what is readily justifiable within the framework of Green's scheme.

The present paper re-examines some of the conclusions of W77, clarifying certain aspects of the spherical polar problem and drawing attention to some overlooked features. Our study is partly prompted by results obtained by Vallis (1982). He found realistic surface zonal flows in a 3-level, zonal average climate model framed in spherical geometry.

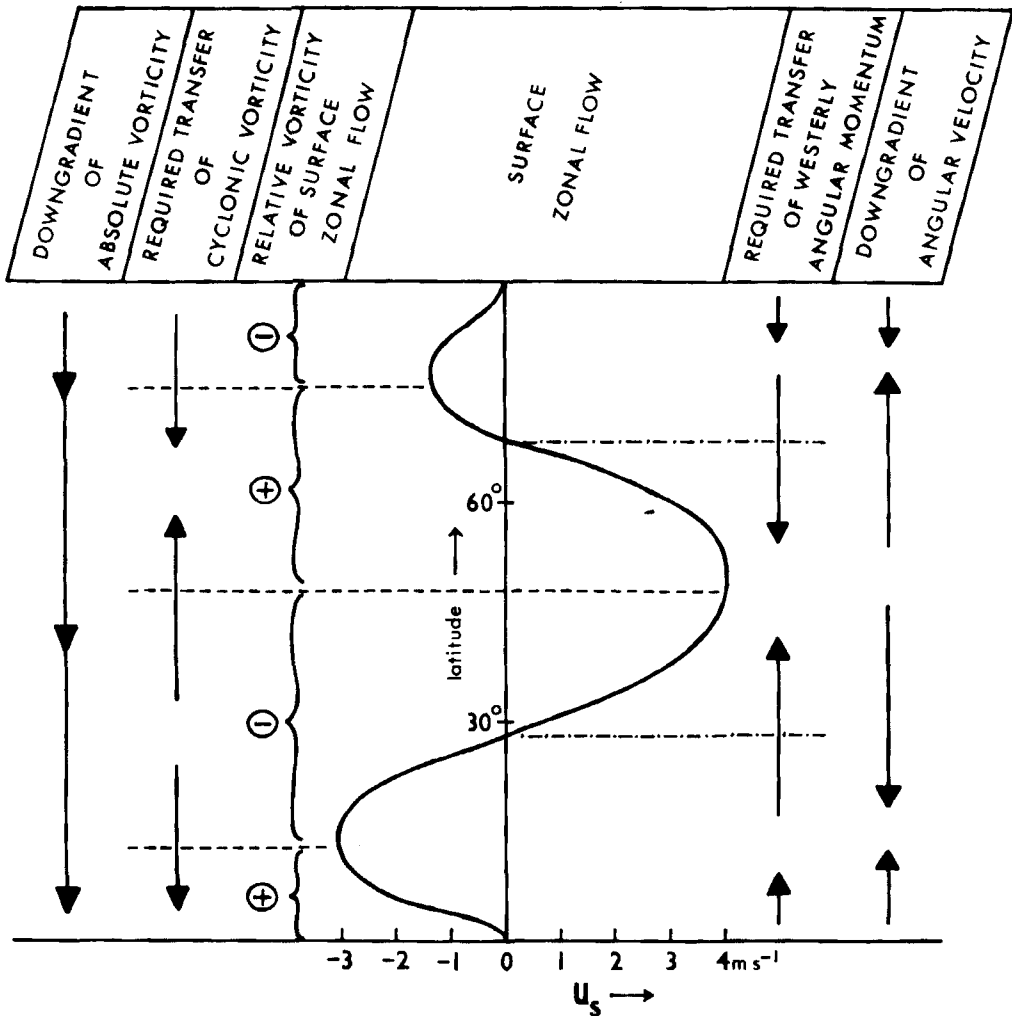


Figure 1. An idealized latitude variation of surface zonal flow U_s . Maximum values are typical of observed tropospheric behaviour. Also shown are the dynamical transfers required to support U_s against the effects of surface friction, and some relevant zonal mean quantities. (The gradients of absolute vorticity and angular velocity indicated are typical of the height-averaged troposphere.) Based on a diagram given in unpublished lecture notes by J. S. A. Green.

Eddy fluxes were represented by Green's parametrizations but without the detailed treatment of spatial variations that W77's results suggested would be necessary.

Following Marshall (1981), a 2-level model is used here instead of the continuous functional representation adopted in W77. This considerably simplifies the analysis, especially as regards the application of a crucial integral constraint on the solution.

In section 2 the derivation of the differential equation for the surface zonal flow U , (as a function of latitude) is outlined. Some useful approximate analytical solutions are considered in section 3, and numerical solutions of the full equation are presented in section 4. The Phillips (1963) type 1 replacements, whereby the Coriolis parameter f is represented by a mid-latitude value, are applied for the most part, but the effects of allowing full latitude variation of f are also investigated. A summary and conclusions are given in section 5.

Zonal mean models based on Green's parametrizations have been used by Sela and Wiin-Nielsen (1971), Wiin-Nielsen and Fuenzalida (1975), Ohring and Adler (1978) and White and Green (1984) as well as Vallis (1982). Marshall (1981) applied the technique in an ocean circulation model. Extensions to three dimensions have been made by White and Green (1982) and Shutts (1983). The relation of mixing theories such as Green's to the various functional extremization approaches of Bretherton and Haidvogel (1976) and Paltridge (see Paltridge, 1981) has been elucidated by Shutts (1981).

The applicability of diffusive parametrizations has of course been widely questioned. Lorenz (1979) found that a diffusion law related observed eddy heat fluxes to zonal mean temperature gradients realistically only at the largest space-scales and on long time-scales. However, in his data analysis Lorenz examined only the simplest law, in which the single diffusion coefficient is assumed constant. Studies of the eddy potential vorticity flux in relation to zonal mean fields (Wiin-Nielsen and Sela 1971; Edmon *et al.* 1980; Pfeffer 1981; Karoly 1982a; Schmitz and Dethloff 1984) reveal predominantly down-gradient transfers. Small regions of up-gradient flux do occur, especially near the tropopause, but typical tropospheric behaviour on seasonal and longer time-scales appears qualitatively consistent with the notion of potential vorticity diffusion.

We consider that dynamical parametrizations of the type proposed by Green offer a sufficiently reasonable idealization of the behaviour of real eddy motion to justify careful study of their implications—the more so because no superior scheme has, to our knowledge, been put forward.

2. EQUATION FOR THE SURFACE FLOW IN A 2-LEVEL MODEL

The dynamical basis of W77 was a type 1 quasi-geostrophic (QG1) model having continuous vertical structure and a standard representation of static compressibility. Here we use a 2-level QG1 model of incompressible flow (although a crude adjustment for the effects of static compressibility will be introduced in section 4).

The vorticity equation is applied at levels 1 and 2 (height $z = H/4, 3H/4$) and the thermodynamic equation at level 2 ($z = H/2$). A smooth, rigid boundary is located at level 4 ($z = H$) and a rough, rigid boundary at level 0 ($z = 0$). The potential vorticity equations are

$$(\partial/\partial t + \mathbf{v}_i \cdot \nabla)q_i = Q_i \quad i = 1, 3 \quad (1)$$

in which \mathbf{v} is the non-divergent geostrophic flow and Q is a (small) source term whose form need not be quoted. The potential vorticities q_i ($i = 1, 3$) are given by

$$q_i = \zeta_i + f + \mu_i^2(\psi_3 - \psi_1) \quad (2)$$

where ψ is the streamfunction corresponding to \mathbf{v} ; $\zeta = \nabla^2\psi$ is the relative vorticity; $\mu_1^2 = -\mu_3^2 = 4f_0^2/gBH^2$, $(gB)^{1/2}$ being the buoyancy frequency at level 2 and f_0 a typical mid-latitude value of $f = 2\omega\sin\theta$ (ω = earth's rotation rate, θ = latitude).

Upon assuming that the zonal average zonal stress vanishes at level 4, it follows from the QG1 momentum equation (see W77, Eq. (43)) applied at levels 1 and 3 that

$$\tau_s = \frac{1}{2}\rho_0 H(\overline{v_1' \zeta_1'} + \overline{v_3' \zeta_3'}) = \frac{1}{2}\rho_0 H(\overline{v_1' q_1'} + \overline{v_3' q_3'}) \tag{3}$$

in the steady state. Here τ_s ($= \overline{\tau}(z = 0)$) is the zonal average zonal stress, v is the poleward component of \mathbf{v} and ρ_0 is the mean fluid density. The overbar and prime signify zonal average and deviation (eddy) components respectively. According to Eq. (3), τ_s is simply related to the poleward eddy fluxes of relative vorticity ζ and potential vorticity q .

Following G70 and W77, the eddy fluxes of q are parametrized in terms of the zonal mean gradients of potential vorticity as

$$\overline{v_i' q_i'} = -(K_i/R) \partial \overline{q_i} / \partial \theta \quad i = 1, 3 \tag{4}$$

in which K_1, K_3 are transfer coefficients and R is the mean radius of the earth. From Eqs. (2), (3) and (4) may be derived a differential equation relating the surface zonal flow U_s to τ_s and the mid-level baroclinicity b ($= -(1/R) \partial \overline{\phi_2} / \partial \theta$, ϕ = log(potential temperature)):

$$\frac{d^2}{d\sigma^2} \{U_s(1 - \sigma^2)^{1/2}\} - \frac{R^2 \tau_s}{\rho_0 H K (1 - \sigma^2)^{1/2}} = 2\omega R - gaH \frac{d^2}{d\sigma^2} \left\{ \frac{b}{f_0} (1 - \sigma^2)^{1/2} \right\} - \frac{\gamma f_0 R^2 b}{BH(1 - \sigma^2)^{1/2}} \tag{5}$$

Here $\sigma = \sin\theta$ and the quantities K, a and γ are given by

$$K = \frac{1}{2}(1 + p)K_1, \quad a = (1 + 3p)/4(1 + p), \quad \gamma = 2(1 - p)/(1 + p), \tag{6}$$

with

$$p = K_3/K_1. \tag{7}$$

Equation (5) is formally similar to Eq. (45) of W77. Our assumptions in solving it are the following.

(a) *The baroclinicity function*

Various analytical forms are chosen for the baroclinicity b (which would of course be determined internally in a full climate model). Each is expressible as

$$b = (\varepsilon \Delta\phi/R)F(\theta) \tag{8}$$

where $F(\theta)$ is the un-normalized functional form, and the constant ε is chosen so that

$$R \int_0^{\pi/2} b d\theta = \Delta\phi. \tag{9}$$

Thus $\Delta\phi$ is the difference in log(potential temperature) between pole and equator at level 2 ($z = H/2$). $\Delta\phi$ is related to the corresponding pole to equator potential temperature difference, ΔT_p , by $\Delta\phi \approx \Delta T_p / \hat{T}_p$ (where \hat{T}_p is a constant reference value). In deriving the numerical solutions presented in section 4 we take $\Delta\phi = 0.154$ in all cases. This corresponds to $\Delta T_p \approx 40$ deg C—a typical northern hemisphere (NH) winter value.

(b) *Latitude variation of K_1 , K_3 and K*

We assume that K_1 and K_3 have the same latitude variation. This is equivalent to assuming separability in latitude and height of the relevant transfer coefficient, and has the convenient consequence that the ratio p (see Eq. (7) and section 2(d)) is independent of latitude. Separability of the transfer coefficient is in broad accord with the results of Wiin-Nielsen and Sela's (1971) diagnostic study (see their Figs. 10 and 12).

The actual latitude variation of K_1 , K_3 is a matter of some uncertainty. From a consideration of baroclinic wave dynamics, G70 argued that K (viewed as a correlation between meridional velocity components and meridional particle displacements) should attain a maximum value in middle latitudes, where the baroclinicity is greatest, and should decrease to very small values in extreme latitudes, where the baroclinicity is small. For a β plane extending between latitudes $y = 0$ and $y = L$, variation as $\sin^2(\pi y/L)$ appears appropriate. Indeed, G70 formalized this expected behaviour in terms of an 'integrative hypothesis'. W77 found that this formalization could not reasonably be extended to the spherical polar problem (see W77, sections 2 and 4). However, variations of the form $\cos^r \theta \sin^s \theta$ (where r and s are small integers) seem to us intuitively reasonable, and the consequences of assuming such forms will be re-examined in section 4(b). We take

$$K = K_m G(\sigma) \quad (10)$$

where

$$K_m = 2.5 \times 10^7 \Delta \phi \text{ m}^2 \text{ s}^{-1} \quad (11)$$

is the mid-latitude maximum value of K . Thus the structure function $G(\sigma)$ attains its maximum value of unity where $K = K_m$. Equation (11) is similar to the form adopted by G70, p. 175, but the constant factor is smaller. This accounts partly for the fact that G70 considered potential temperature differences between 20 and 70°N; but the choice (11) has been made primarily to ensure that, when $\Delta \phi \approx 0.15$, K_m takes a value similar to those observed in the NH winter troposphere ($\approx 4 \times 10^6 \text{ m}^2 \text{ s}^{-1}$; see Wiin-Nielsen and Sela (1977) and Sela and Wiin-Nielsen (1971)). In W77, this value was adopted for the latitude average of K , thus giving an overestimate by a factor of about two.

(c) *Surface stress law*

The surface stress τ_s is assumed expressible in terms of the surface zonal flow U_s . As in G70 and W77 we take

$$\tau_s = \rho_0 k U_s \quad (12)$$

where k is a friction-layer parameter. Equation (5) can then be re-written in terms of the dependent variable $U = U_s(1 - \sigma^2)^{1/2}$ as

$$\frac{d^2 U}{d\sigma^2} - \frac{\nu^2 U}{(1 - \sigma^2)} = 2\omega R - gaH \frac{d^2}{d\sigma^2} \left\{ \frac{b}{f_0} (1 - \sigma^2)^{1/2} \right\} - \frac{\gamma f_0 R^2 b}{BH(1 - \sigma^2)^{1/2}}, \quad (13)$$

in which

$$\nu^2 = kR^2/HK. \quad (14)$$

For the friction-layer parameter k we adopt the value $2 \times 10^{-2} \text{ m s}^{-1}$, independent of latitude. This is somewhat larger than the value assumed in W77, but is consistent with the surface zonal stress and zonal flow determinations reviewed by White (1974).

With the above values of k , K_m and $\Delta\phi$, and $R = 6.4 \times 10^6 \text{m}$, $H = 10^4 \text{m}$, we obtain

$$\nu^2 = 22/G(\sigma) \quad (15)$$

where $G(\sigma)$ is the structure function introduced in Eq. (10).

(d) *Boundary conditions and surface torque constraint*

As in W77, boundary conditions on U_s are applied as

$$U_s = 0 \quad \text{at } \sigma = 0, 1. \quad (16)$$

and the constraint

$$\int_0^1 U_s (1 - \sigma^2)^{1/2} d\sigma = \int_0^1 U d\sigma = 0 \quad (17)$$

is also imposed. Constraint (17) represents the vanishing of the total surface torque between solid earth and atmosphere in the steady state, and it is obeyed by requiring the constant p (see Eqs. (6) and (7)) to take the appropriate value. Thus p is determined by the model, and does not have to be specified externally.

Our 2-level model is more consistent than the continuous vertical structure formulation used in W77. The quantities a and γ are here both determined via Eqs. (6) once p has been found by applying constraint (17); whereas in W77 constraint (17) was obeyed by varying only the equivalent of γ , and the equivalent of a was specified externally. In fact, the term in a is of minor importance in tropospheric parameter ranges, and W77's procedure amounts to only a small quantitative inconsistency. The most important new aspects of this study are investigations of the effects of different choices for the baroclinicity b and the latitude structure of the transfer coefficient K (as it appears in the quantity ν^2 —see Eqs. (14) and (15)). We also examine how results are changed when f_0 in Eq. (13) is replaced by $f = 2\omega \sin\theta = 2\omega\sigma$.

Equation (13) is solved numerically by dividing the interval $x = [0, 1]$ into N_I equal intervals and using the Gaussian algorithm, or a recurrence method, to obtain the discrete solution which obeys conditions (16) and (17). For further details see Wu (1983). We take $N_I = 18$; higher resolutions lead to only very small changes. In section 3, however, we consider some useful analytical solutions of the problem.

3. ANALYTICAL SOLUTIONS

If the term in τ_s is neglected, Eq. (5) can be written as

$$\frac{d^2}{d\sigma^2} \left\{ \left[U_s + \frac{gaH}{f_0} b \right] (1 - \sigma^2)^{1/2} \right\} = 2\omega R - \frac{\gamma f_0 R^2 b}{BH(1 - \sigma^2)^{1/2}}. \quad (18)$$

Analytical solution of Eq. (18) using the procedure outlined in section 2 is straightforward. With $f_0 = \sqrt{2} \cdot \omega$, and a particular choice of baroclinicity, $(U_s/\omega R)$ depends only on the non-dimensional parameters

$$\delta_1 = \Delta\phi/BH \quad (19)$$

$$\delta_2 = gH\Delta\phi/\omega^2 R^2. \quad (20)$$

The parameter δ_1 is the ratio of the potential temperature differences between pole and equator, and between bottom ($z = 0$) and top ($z = H$) of the model troposphere. δ_2 is a Rossby number based on the thermal wind shear and the absolute velocity of rotation ωR of a point fixed on the earth at the equator. Typical tropospheric values of δ_1 and δ_2 are 1 and 0.05.

Solutions in this zero-stress approximation should be regarded as representing the limit of very weak surface stress, for the constraint (17) cannot be justified if τ_s vanishes completely. As was discussed in W77, such solutions are useful guides to the forms of solutions of the complete equation (5) although they invariably overestimate extremal values. Here, some solutions of Eq. (18) will be presented to indicate the consequences of making various changes in the conditions applied in solving Eq. (5). No algebraic

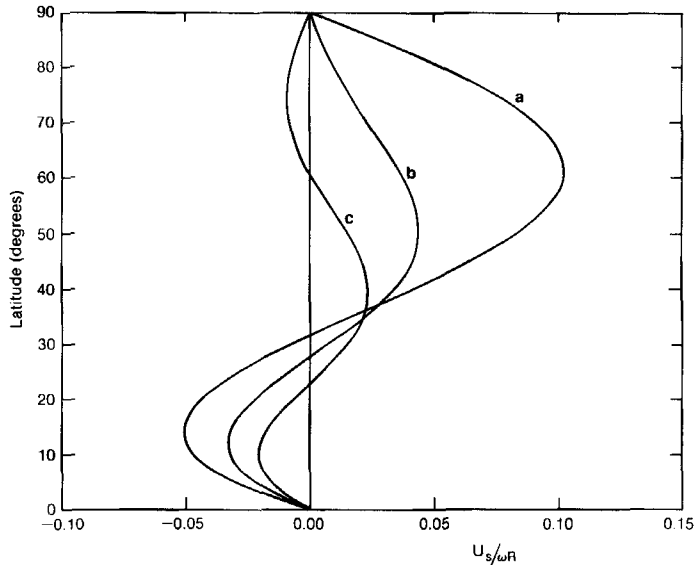


Figure 2. Latitude variations of surface zonal flow (expressed as $U_s/\omega R$) obtained by solving Eq. (18) analytically, assuming various forms for the baroclinicity. Curves (a), (b), (c) correspond respectively to the baroclinicities given by Eqs. (21), (22), (23)—whose latitudes of maximum horizontal temperature gradient are in decreasing order.

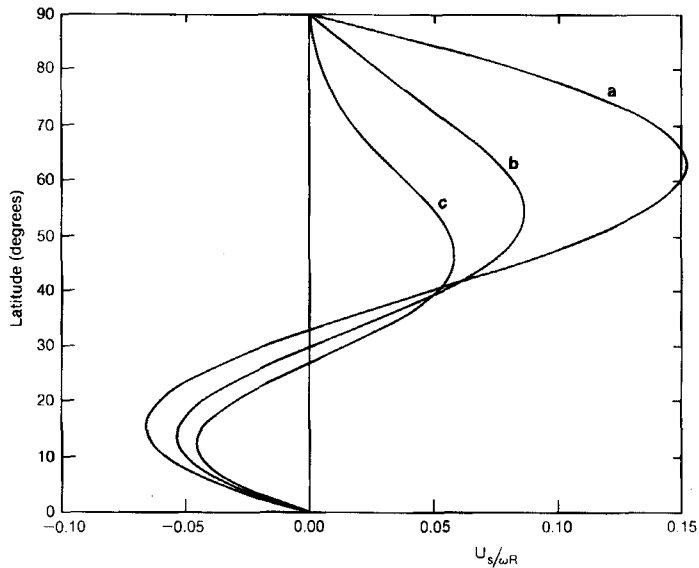


Figure 3. As Figure 2, but showing the results obtained when $f_0 = \sqrt{2} \cdot \omega$ (in Eq. (18)) is assigned the true latitude variation of the Coriolis parameter.

results will be quoted, and all physical interpretation will be deferred until section 4. (Note, however, that our choices for the baroclinicity in this section are dictated largely by a desire to avoid algebraically complicated results. Our objective is to demonstrate qualitative effects only.)

An important feature of any assumed baroclinicity is the latitude θ_{\max} of its maximum. Figure 2 shows the variations of $U_s/\omega R$ obtained by solving Eq. (18) when $\delta_1 = 1$, $\delta_2 = 0.05$ and

$$b = (3\Delta\phi/R)\sin^2\theta\cos\theta \quad \theta_{\max} = 54^\circ 44' \quad (21)$$

$$b = (15\Delta\phi/2R)\sin^2\theta\cos^3\theta \quad \theta_{\max} = 39^\circ 14' \quad (22)$$

$$b = (105\Delta\phi/8R)\sin^2\theta\cos^5\theta \quad \theta_{\max} = 32^\circ 18'. \quad (23)$$

Moving the latitude of maximum baroclinicity equatorwards clearly has two effects: (i) to reduce extremal values; and (ii) ultimately to produce polar easterlies. Qualitatively similar conclusions may be reached by representing the baroclinicity by a δ function centred at $\theta = \theta_{\max}$ (White 1974).

Figure 3 shows the variations of $U_s/\omega R$ obtained by solving Eq. (18) with the same choices of baroclinicity, Eqs. (21)–(23), and the same values of δ_1 and δ_2 , but with $f_0 = \sqrt{2}$. ω replaced by $f = 2\omega\sigma$ (the true latitude variation of the Coriolis parameter). It is evident that equatorward movement of θ_{\max} leads to marked reductions of extremal values—as in the previous case—but polar easterlies do not appear even when $\theta_{\max} = 32^\circ 18'$. Comparison of corresponding curves in Figs. 2 and 3 suggests another important result: putting $f = 2\omega\sigma$ instead of $f = f_0 = \sqrt{2}$. ω in Eq. (18) leads to marked increases of extremal values.

4. NUMERICAL SOLUTIONS

Interpretation of results is facilitated by writing Eq. (5) in terms of τ_s :

$$\tau_s = -\frac{\rho_0 HK(1-\sigma^2)^{1/2}}{R^2} \left\{ 2\omega R - gaH \frac{d^2}{d\sigma^2} \left[\frac{b}{f_0} (1-\sigma^2)^{1/2} \right] - \frac{d^2}{d\sigma^2} [U_s(1-\sigma^2)^{1/2}] - \frac{\gamma f_0 R^2 b}{BH(1-\sigma^2)^{1/2}} \right\}. \quad (24)$$

The terms on the right-hand side represent various components of the height-integrated potential vorticity flux, as expressed by Green's transfer equations (see Eqs. (3) and (4)). The first three terms are essentially barotropic in character: they represent components of the height-integrated absolute vorticity gradient. The planetary vorticity gradient term is usually the largest of these barotropic terms (and the first of the second derivative terms reinforces it over wide ranges of latitude given a realistic baroclinicity, b). Thus the net effect of the barotropic terms is to give negative values of τ_s , and hence easterly surface flow. The remaining r.h.s. term in Eq. (24) represents baroclinic effects. It tends to give positive values of τ_s , and hence westerly surface flow, if $\gamma > 0$ and $b > 0$. Since the total stress torque over the northern hemisphere must vanish in the present model in the steady state (see Eq. (17)), the quantity γ —here assumed independent of latitude—must be positive when the temperature on level surfaces increases monotonically from pole to equator ($b \geq 0$). Thus surface westerlies are most likely to occur in middle latitudes, where b attains a maximum; see G70. In low latitudes surface easterlies are expected. In high latitudes the dominance of barotropic and baroclinic effects is unclear: all the barotropic terms are subject to the factor $(1-\sigma^2)^{1/2}$, but this factor cancels from the baroclinic term.

According to Eqs. (6) and (7), positive γ corresponds to a bias of the transfer coefficients K_i towards the lower level of the 2-level atmosphere, and thus to the steering level of baroclinic waves being below $z = H/2$.

(a) *Dependence of solutions on the form of the baroclinicity*

Let us neglect the latitude variation of the transfer coefficients K_i . Taking $G(\sigma) = 1$ in Eq. (15) we then obtain $\nu^2 = 22$. Curve (a) in Fig. 4 shows the solution $U_s(\theta)$ which results in this case when

$$b = (2\Delta\phi/R)\sin\theta\cos\theta. \tag{25}$$

There are no polar easterlies, and easterly and westerly maxima are 8.7 and 12.5 ms^{-1} respectively. As noted in W77 for a similar solution (see Fig. 10 of that paper), these maxima are much larger than observed tropospheric values (see, for example, Fig. 1, above). However, the baroclinicity given by Eq. (25) attains a maximum at $\theta = 45^\circ$ and is symmetric about that latitude. In the northern hemisphere winter, the baroclinic zone is located between 20 and 50°N ; see Fig. 5(a). The form

$$b = (3\Delta\phi/R)\sin\theta\cos^2\theta \tag{26}$$

has a maximum close to 35°N , and better represents the observed NH winter variation (see Fig. 5(b)). The analytical results of section 3 suggest that the form (26) will give reduced surface flow intensities, as compared with the form (25), because it has maximum baroclinicity nearer the equator. Figure 4, curve (b), shows the solution $U_s(\theta)$ obtained by solving Eq. (13) with $\nu^2 = 22$, as before, but with b represented by Eq. (26). Marked reductions of intensity are indeed seen: maximum easterlies and westerlies are 6.9 and 7.3 ms^{-1} respectively. In physical terms, the equatorward movement of the baroclinic zone serves to cancel out a considerable part of the barotropic vorticity transfer in low/

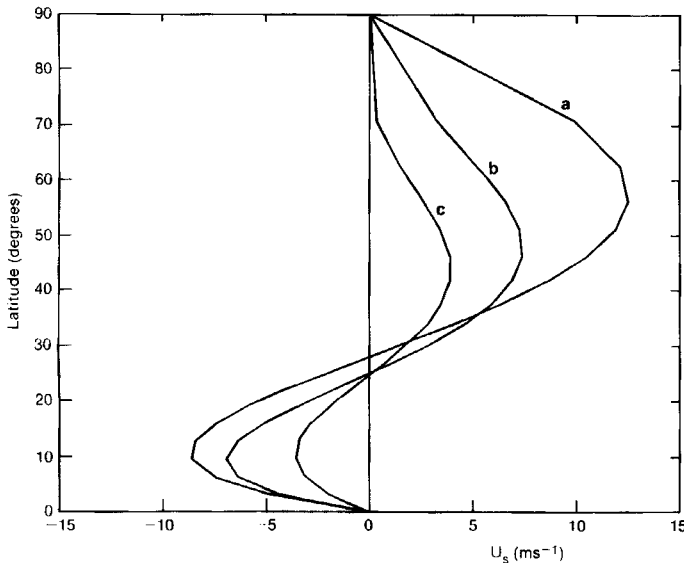


Figure 4. Latitude variations of surface zonal flow U_s obtained by solving Eq. (13) numerically under various assumptions. Curve (a): $\nu^2 = 22$ (independent of latitude), baroclinicity given by Eq. (25); curve (b): $\nu^2 = 22$, baroclinicity given by Eq. (26); curve (c): as for curve (b), but with latitude variation of ν^2 permitted according to Eq. (31) (which includes an adjustment to allow for the effect of static compressibility). $\Delta\phi = 0.154$ in each case.

middle latitudes, and requires a weaker baroclinic vorticity transfer in high latitudes than before. Thus both easterlies and westerlies are weakened.

The effect of moving the baroclinic zone equatorward was not noted in W77. In fact, it was incorrectly stated (p. 111) that "solutions . . . are not sensitive to the choice of b ". Here we have found significant reductions in extremal values when the form (26), rather than (25), is used to represent b . A further example—involving a greater change in b —will be considered in section 4(c).

(b) *Dependence of solutions on the latitude variation of the transfer coefficient*

In W77 it was found that considerable reductions in extremal values of U_s could be obtained by allowing for the latitude variation of the transfer coefficient K . The assumed variation was of the form $\sigma^2(1 - \sigma^2)$, and the latitudinal mean value of K was held constant in the comparison. This is a reasonable adjustment (see G70), but, as noted in section 2(b), the mean value adopted was equal to observed tropospheric mid-latitude maxima ($\approx 4 \times 10^6 \text{ m}^2 \text{ s}^{-1}$) and was thus too high by a factor of two.

Here we investigate the effect of applying the structure function

$$G(\sigma) = \{(3\sqrt{3})/2\}\sigma(1 - \sigma^2) \quad (27)$$

to the K_i (see Eq. (10)). Thus the K_i retain their previous values at the maximum latitude

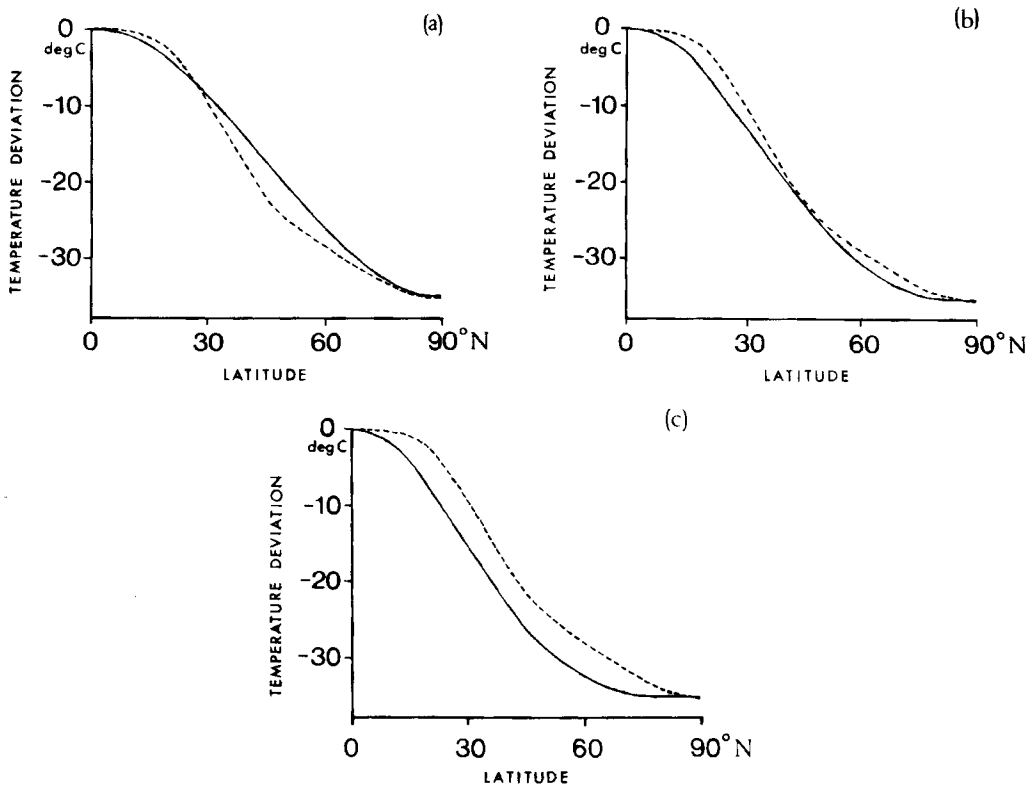


Figure 5. (a)–(c). Comparison of observed latitude variation of zonal mean temperature at 500 mb in January with three analytic forms. Dotted lines: observed variation (for $\theta < 75^\circ \text{N}$, from Oort and Rasmusson (1971); for $75^\circ \leq \theta \leq 85^\circ \text{N}$, calculated from data of Lejenas and Madden (1982) (P. Nalpanis, private communication)). Full lines: variations implied by (a) Eq. (25), (b) Eq. (26), (c) Eq. (32); the amplitude factor has been chosen in each case to give the same extreme temperature difference as the observed value. All temperatures are defined relative to an arbitrary zero at the equator.

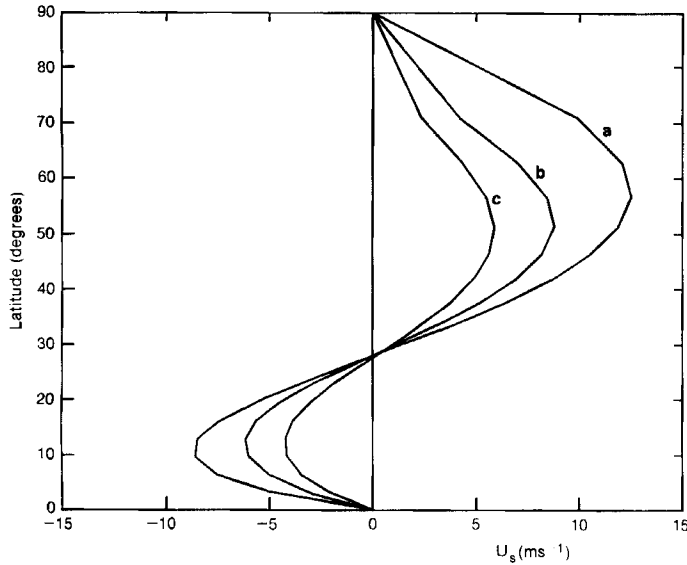


Figure 6. Latitude variations of surface zonal flow U_s obtained by solving Eq. (13) numerically under various assumptions. Curve (a): $\nu^2 = 22$ (independent of latitude), baroclinicity given by Eq. (25); curve (b): $\nu^2 = 22/G(\sigma)$, with $G(\sigma)$ given by Eq. (27), baroclinicity given by Eq. (25); curve (c): as for curve (b), but with ν^2 scaled so as to allow crudely for the effects of static compressibility (see Eq. (31)). $\Delta\phi = 0.154$ in each case.

($35^{\circ}16'$) only, and are decreased at all other latitudes. Figure 6, curve (b), shows $U_s(\theta)$ obtained by solving Eq. (13) with b given by Eq. (25) and $\nu^2 = 22/G(\sigma)$; curve (a) shows the corresponding solution when $\nu^2 = 22$ (given above as curve (a) in Fig. 4). Easterly and westerly maxima are reduced to 6.1 and 8.8 m s^{-1} . These values represent considerable reductions (and are comparable to those found in section 4(a)). Even greater reductions are obtained if K is assumed to vary as $\sigma^2(1 - \sigma^2)$, as in W77. If the grounds for comparison are those adopted here, curve A of W77's Fig. 10 and curve B of W77's Fig. 11 correspond to one another; a reduction of easterly/westerly maxima from $9/13 \text{ m s}^{-1}$ to $4/7 \text{ m s}^{-1}$ is seen.

(c) Effects of accounting for static compressibility

Static compressibility has been neglected so far in this study. It was represented in W77, however, and may easily be accounted for in our 2-level formulation. With $\rho = \rho_i$ at level i , the counterpart of Eq. (13) turns out to be

$$\frac{d^2 U}{d\sigma^2} - \frac{\mu^2 U}{(1 - \sigma^2)} = 2\omega R - g\hat{a}H \frac{d^2}{d\sigma^2} \left\{ \frac{b}{f_o} (1 - \sigma^2)^{1/2} \right\} - \frac{\hat{\gamma} f_o R^2 b}{BH(1 - \sigma^2)^{1/2}}. \tag{28}$$

Here

$$\left. \begin{aligned} \hat{a} &= (1 + 3\hat{p})/4(1 + \hat{p}) \\ \hat{\gamma} &= 2(\rho_2 - \rho_3\hat{p})/\rho_1(1 + \hat{p}) \\ \mu^2 &= \rho_s k R^2 / \bar{\rho}^z H \bar{K} \end{aligned} \right\}$$

with

$$\bar{K} = (\rho_1 K_1 + \rho_3 K_3) / 2\bar{\rho}^z \tag{29}$$

$$\rho_s = \rho(z = 0)$$

$$\bar{\rho}^z = (\rho_1 + \rho_3) / 2$$

and

$$\hat{p} = \rho_3 K_3 / \rho_1 K_1$$

Equation (28) can be solved numerically by the same methods as was Eq. (13). (An isothermal reference atmosphere, with a reasonable scale height, may be assumed in order to determine the densities ρ_i .) It is found that the solutions differ from those obtained by solving Eq. (13), in corresponding cases, almost entirely through the differences between the quantities μ^2 and ν^2 . Since \bar{K} (see Eq. (29)) is a mass-weighted average of K_1 and K_3 , it is equivalent to K , as given by Eq. (6), in the incompressible case. Thus

$$\mu^2 = \rho_s \nu^2 / \bar{\rho}^2. \quad (30)$$

This difference can be allowed for by solving Eq. (13) with the term in ν^2 suitably scaled up. A convenient way of doing this is to determine K_1 , rather than K , from Eq. (10); the effect is to increase ν^2 by a factor of $2/(1+p)$, which (somewhat fortuitously) is almost equal to typical values of $\rho_s/\bar{\rho}^2$. To take account of static compressibility we shall in the remainder of this paper follow the procedure of replacing the term ν^2 in Eq. (13) by $2\nu^2/(1+p)$. Figure 6, curve (c), shows the solution obtained when the case previously examined in section 4(b), with baroclinicity given by Eq. (25), is treated in this way: the term in ν^2 in Eq. (13) now takes the form

$$-44U/\{(1+p)G(\sigma)(1-\sigma^2)\} \quad (31)$$

(with $G(\sigma)$ given by Eq. (27), as before). Westerly and easterly maxima are now reduced to 5.9 and 4.2 m s^{-1} respectively. Figure 4, curve (c), shows the solution obtained using the adjustment (31) when b is represented by Eq. (26) (the more realistic form). Extremal values are now 4.0 and -3.5 m s^{-1} . These are similar to typical observed values (see Fig. 1), though weak westerlies ($<0.4 \text{ m s}^{-1}$), rather than easterlies, here occur poleward of 70° . The extremal values are similar also to those obtained by Vallis (1982) in a 3-level, pressure coordinate, zonal average climate model which used Green's (1970) parametrization scheme—see his Fig. 5. In Vallis's model, the baroclinicity was generated internally, a fairly detailed treatment of diabatic effects being incorporated. Full latitude variation of the Coriolis parameter was allowed in the explicit formulation, but QG1 approximations were applied in the eddy flux parametrizations (and thus largely determined the surface zonal flow).

Since Vallis obtained weak surface easterlies ($<1 \text{ m s}^{-1}$) poleward of 80° , we have derived some further solutions of Eq. (7) to investigate the circumstances in which polar easterlies may occur in our model. The arguments given at the beginning of this section suggest that polar easterlies will appear when the assumed baroclinicity is sufficiently small in high latitudes that the barotropic terms in Eq. (23) become dominant there. The form

$$b = (4\Delta\phi/R)\sin\theta\cos^3\theta \quad (32)$$

has a maximum at 30°N (see Fig. 5(c)). Figure 7 shows the solution obtained using Eq. (32) when static compressibility is allowed for as in (31). A broad band of weak polar easterlies—extending as far south as 60°N and with a maximum of about 0.8 m s^{-1} —is produced, and easterly/westerly maxima are both about 3.0 m s^{-1} . The polar easterlies are thus much more extensive than in Vallis's result. Although the assumed form of b is somewhat unrealistic (see Fig. 5(c)), we feel that the solution shown in Fig. 7, as compared with curve (c) of Fig. 4, is sufficient to demonstrate the sensitivity of the high latitude surface zonal flow to changes in the form of the baroclinicity (and hence to other modelling assumptions). This sensitivity is suggested also by the analytical solutions presented in section 3. Thus we consider that the differences between our results and

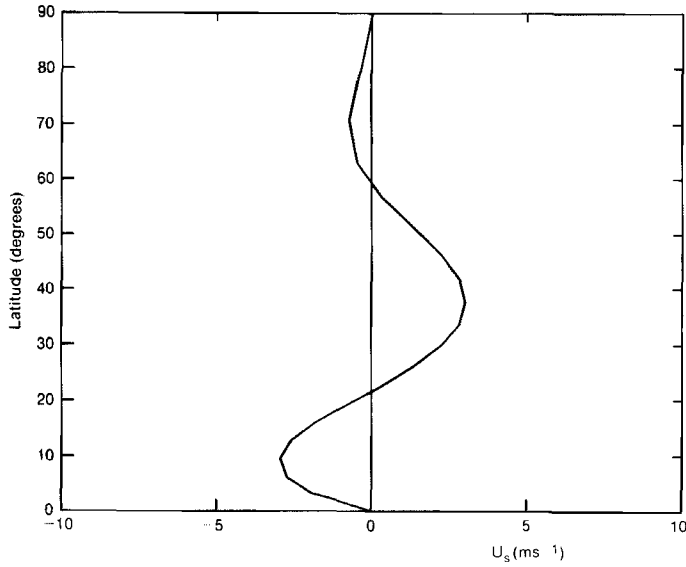


Figure 7. Latitude variation of surface zonal wind U_s obtained by solving Eq. (13) numerically when the baroclinicity is given by Eq. (32). Latitude variation of ν^2 as in Eq. (31) (static compressibility allowed for). $\Delta\phi = 0.154$.

Vallis's are of little significance, given the differences in formulation between the two models.

(d) *Effects of allowing latitudinal variation of the Coriolis parameter*

In the QG1 model the Coriolis parameter f appears as the mid-latitude value f_0 (here taken as $\sqrt{2} \cdot \omega$) in the vortex stretching term of the vorticity equation and also in the thermal wind equation. Thus f_0 appears in the second and third terms on the r.h.s. of Eq. (13). Although allowing $f = 2\omega\sigma$ instead of $f = f_0$ in these terms is strictly inconsistent because of the consequent loss of conservation properties in the basic formulation (see W77, p. 115), it seems reasonable to expect that such variations should give a more accurate representation of behaviour on the sphere. (Similar variable f replacements have been made by Edmon *et al.* (1980), Karoly (1982b) and Andrews (1983), amongst others, in various studies of large-scale atmospheric motion.) To investigate this possibility we have solved Eq. (13) with and without the indicated replacements. With $f = 2\omega\sigma$, the r.h.s. of Eq. (13) becomes

$$2\omega R - \frac{gaH}{2\omega} \frac{d^2}{d\sigma^2} \left\{ \frac{b(1 - \sigma^2)^{1/2}}{\sigma} \right\} - \frac{2\gamma\omega R^2 b\sigma}{BH(1 - \sigma^2)^{1/2}} \tag{33}$$

We consider the case in which

$$b = (15\Delta\phi/2R)\sin^2\theta \cos^3\theta \tag{34}$$

and

$$\nu^2 = 44/\{(1 + p)G(\sigma)\} \tag{35}$$

(with $G(\sigma)$ as given by Eq. (27)). In Fig. 8, curve (a) is the result obtained when $f = f_0$; curve (b) is the result when $f = 2\omega\sigma$ only in the γ term in (33); curve (c) is that when $f = 2\omega\sigma$ in both the γ and a terms. Clearly, allowing the full latitude variation of the

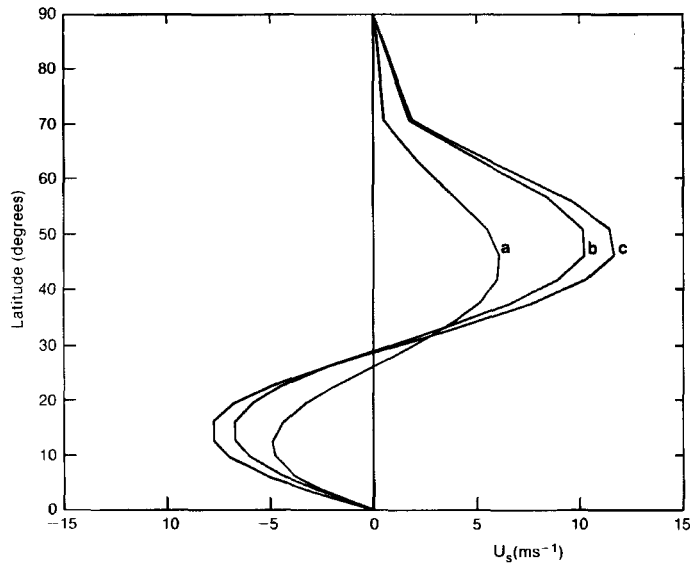


Figure 8. Effect of latitude variation of the Coriolis parameter f on the calculated surface zonal flow $U_s(\theta)$. Solutions obtained numerically, baroclinicity given by Eq. (34), v^2 given by Eq. (35) (static compressibility allowed for). Curve (a) shows the solution obtained from Eq. (13), in which the usual $f = f_0$ replacements of the QG1 formulation are made; curve (c) shows the solution obtained when $f = 2\omega\sigma$ is retained throughout; curve (b) shows an intermediate case. See text for further details. $\Delta\phi = 0.154$ in each case.

Coriolis parameter considerably increases extremal values, and polar easterlies do not appear: easterly and westerly maxima in the $f = f_0$ case are 4.9 and 6.0 m s^{-1} , increasing to 7.8 and 11.6 m s^{-1} in the $f = 2\omega\sigma$ case. These results are in accord with the indications of the analytical solutions presented in section 3. To obtain reasonable surface zonal flows it is evidently necessary to assume variations of the K_i which are far more peaked in middle latitudes (as first suggested by White (1974)). Physical interpretation is straightforward. The effect of allowing $f = 2\omega\sigma$ is to decrease baroclinic vorticity transfers equatorward of 45° and to increase them poleward of 45° . Hence both easterly and westerly surface wind maxima are markedly increased, and the tendency for polar easterlies to appear is less than before.

5. CONCLUDING REMARKS

In the atmosphere, mean meridional circulations and stationary eddies, in addition to transient eddies, transfer angular momentum meridionally. The stationary eddies transfer only one quarter to one third of the total angular momentum flux (see Oort and Rasmusson 1971), and some of this stationary eddy transfer must be contributed by essentially transient eddies which are localized in longitude by the stationary long-wave pattern. The theory developed in G70 should therefore account for nearly all the eddy flux of angular momentum. In middle and high latitudes the flux by meridional circulations is negligible in the height average (which is the proper consideration in the surface flow problem). Meridional circulations are quite important in the height-integrated angular momentum balance in very low latitudes ($<10^\circ$ —see Kidson *et al.* (1969)); but White (1974) found that the calculated surface zonal flow was only slightly changed when the contribution of the meridional circulation was allowed for in solving the relevant equation (equivalent to our Eq. (5)).

The present study has confirmed W77's finding that representing the expected smallness of the transfer coefficient K in extreme latitudes reduces the intensity of the calculated surface zonal flow. However, because we have specified a more realistic latitude average of K , predicted numerical values of U_s are here somewhat less than those found in W77.

Use of a more realistic representation of the zonal average baroclinicity than in W77 also contributes a noticeable reduction of calculated surface zonal flow intensities. Indeed, the surface zonal flow has a realistic intensity when both of the above effects are suitably represented and the latitude variation of the Coriolis parameter is approximated in the prescribed manner of the QG1 formulation. Because the zone of subtropical easterlies is somewhat reduced, the implied latitudinal extent of the Hadley cell and equatorward penetration of the Ferrel cell are also more realistic. These results are consistent with those obtained by Vallis (1982) using similar QG1 parametrizations in a zonal average climate model which differs from ours in several respects. However, Vallis obtained weak easterlies in high latitudes where we obtain weak westerlies. It appears that the balance which determines the high-latitude surface flow in Green's scheme is very delicate; polar easterlies or polar westerlies may be produced depending on details of model formulation and behaviour.

Contrary to the implications of W77's results, we conclude therefore that Green's parametrization scheme can produce realistic surface flows on the sphere so long as QG1 dynamics are assumed. Nevertheless, and as stated in W77, allowing the full latitude variation of the Coriolis parameter leads to a considerable increase in the intensity of the calculated surface zonal flow, and the tendency for polar easterlies to appear is much reduced. The formulation used is then not strictly consistent dynamically (because it does not imply the good integral conservation properties of the QG1 model) but it may be considered to be more accurate than QG1. In this case, Green's parametrization scheme, as at present formulated, does not pass the surface flow test. The production of a realistic surface zonal flow would evidently require a more detailed specification of the latitude and height variation of the relevant transfer coefficient—to an extent which the theory is not yet equipped to justify. It is a challenge for future work to establish good theoretical grounds for the required spatial variations.

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